OUTPUT-FEEDBACK NONLINEAR MODEL PREDICTIVE CONTROL USING HIGH-GAIN OBSERVERS IN ORIGINAL COORDINATES

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Abstract

In the recent years various nonlinear model predictive control (NMPC) schemes have been derived that achieve guaranteed stability of the closed loop. However, most of these schemes are based on the assumption that the full state information is available. Since in general not all states can be directly measured, it is of paramount importance how one can estimate the not directly measurable states without jeopardizing the stability of the closed-loop. To overcome this problem it has been shown in recent works, that combining sampled-data NMPC for continuous time systems with high-gain observers can lead to semi-global practical stability. However, the resulting output feedback controller uses a high-gain observer which is formulated in observability normal form. Thus explicit knowledge of the inverse of the observability map is required. In this paper we show that one can also use high-gain observers in original coordinates, thus allowing to circumvent the explicit knowledge of the inverse of the observability map. The resulting output feedback schemes is applied to the control of a mixed culture bioreactor.

1 Introduction

Model predictive control for systems described by nonlinear ODEs or difference equations has received considerable attention over the past years. Several schemes that guarantee stability in the state feedback case exist by now, see for example [1, 5, 16] for recent reviews. Much fewer results are available in the case when not all states are directly measured. To overcome this problem, often a state observer together with a stabilizing state feedback NMPC controller is used. However, due to the lack of a general nonlinear separation principle, the stability of the resulting closed loop must be examined separately.

Several researchers have addressed the output feedback NMPC problem. The approach in [6] derives local uniform asymptotic stability of contractive NMPC in combination with a “sampled” state estimator. In [14, 15], see also [19], asymptotic stability results for observer based discrete-time NMPC for “weakly detectable” systems are given. The results allow, in principle, to estimate a (local) region of attraction of the output feedback controller from Lipschitz constants. However, it is in general not clear which parameters in the state feedback controller and observer should be changed to increase the region of attraction, or how to recover the region of attraction of the state feedback controller. In contrast to these approaches, the control strategies derived in [7, 8, 12, 18] establish semi-global stability results, delivering direct tuning knobs to increase the resulting region of attraction of the closed-loop. The approach presented in [18] consist of an optimization based moving horizon observer combined with the so called dual-mode NMPC scheme proposed in [17].

In [12] asymptotic stability for instantaneous NMPC using high-gain observers for state recovery is obtained. The drawback is that the open-loop optimal control problem appearing in the NMPC controller must be solved at every time instant. To avoid this the results have been expanded in [7, 8] to the general sampled-data NMPC case, showing that the closed-loop is semi-global practically stable. In sampled-data NMPC the open-loop optimal control problem is only solved at discrete sampling instants and the resulting optimal input signal is applied open-loop in between.

While the results given in [8] allow to consider a wide class of systems, assuming that a uniform observability assumption holds, in its basic formulation they require the observer to be implemented in observability normal-form coordinates. Often this is difficult, since one has to find the explicit inverse of the observability map to transfer the resulting state estimates back to the original coordinates. In this paper we show, based on the results given in [4, 13], that the observer can be also designed in original coordinates. Using a suitable DAE integrator, this allows to avoid the explicit knowledge of the inverse of the observability map and thus simplifies the implementation of the controller.

The paper is structured as follows: In Section 2 we review the NMPC output feedback scheme given in [8]. Based on this we show in Section 3 how one can design high-gain observers that do not require the explicit knowledge of the inverse of the observability map. The resulting output feedback scheme is employed in Section 4 to the control of a mixed culture bioreactor.

2 Semi-Globally Practically Stable Output-Feedback NMPC using High-Gain Observers

We consider nonlinear systems given by

$$\dot{x} = f(x,u), \qquad y = h(x) \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ denotes the system state, $u \in \mathcal{U} \subset \mathbb{R}^m$ is the system input, $y \in \mathbb{R}^p$ is the measured output, and $\mathcal{X}, \mathcal{U}$ denote the constrained sets of allowed states and inputs. The sets $\mathcal{X}$ and $\mathcal{U}$ are such that $\mathcal{U} \subset \mathbb{R}^p$ is compact, $\mathcal{X} \subseteq \mathbb{R}^n$ is connected and $(0,0) \in \mathcal{X} \times \mathcal{U}$. With respect to the functions $f : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}^p$ we assume that they are sufficiently smooth. Furthermore, the origin is as stationary point, i.e. $f(0,0) = 0$ and $h(0) = 0$.

We consider uniform complete observable systems. Uniform complete observability is defined in terms of the observability map $\mathcal{H}$, which is given by the successive differentiation of the output y:

$$\begin{aligned} Y^T &= \left[y_1, \dot{y}_1, \ldots, y_1^{(r_1)}, y_2, \ldots, y_p, \ldots, y_p, \ldots, y_p^{(r_p)}\right] \\ &=: \mathcal{H}(x,U)^T \end{aligned}$$

Here Y is the vector of output derivatives, U contains the input and all input derivatives that appear. We assume that the system is uniform complete observable and, for simplicity of presentation, we furthermore assume that $\mathcal{H}$ is independent of the derivatives of the system input, i.e. $U = u$[1].

Assumption 1 (Uniform Complete Observability without Input Derivatives): *The system* (1) *is uniformly completely observable in the sense that there exists a set of indices* $\{r_1, \ldots, r_p\}$ *such that the mapping* $Y = \mathcal{H}(x,u)$ *depends only on* x *and* u*, is smooth with respect to* x *and its inverse from* Y *to* x *is smooth and onto for any* u.

2.1 NMPC State Feedback

In the framework of predictive control, the input is defined by the solution of an open-loop optimal control problem that is solved at sampling instants. Between the sampling instants the optimal input is applied open-loop. For simplicity we denote the sampling instants by t_i, with $t_i - t_{i-1} = \delta$, δ being the sampling time. For a given t, t_i should be taken as the nearest sampling instant $t_i < t$. The open-loop optimal control problem solved at any t_i is given by:

$$\min_{\bar{u}(\cdot)} J(\bar{u}(\cdot); x(t_i)) \tag{2a}$$

$$\text{subject to:} \quad \dot{\bar{x}} = f(\bar{x}, \bar{u}), \quad \bar{x}(\tau = 0) = x(t_i) \tag{2b}$$

$$\bar{u}(\tau) \in \mathcal{U}, \;\; \bar{x}(\tau) \in \mathcal{X} \;\; \tau \in [0, T_p] \tag{2c}$$

$$\bar{x}(T_p) \in \mathcal{E}. \tag{2d}$$

The cost functional J is defined over the control horizon T_p by the stage cost F and the terminal penalty E.

$$J(\bar{u}(\cdot); x(t_i)) := \int_0^{T_p} F(\bar{x}(\tau), \bar{u}(\tau)) d\tau + E(\bar{x}(T_p)).$$

Instead of going into details on the different stabilizing state feedback NMPC schemes, we merely assume in the following that the NMPC scheme fits into the given frame and satisfies the following assumptions.

Assumption 2 *There exists a simply connected region* $\mathcal{R} \subseteq \mathcal{X} \subseteq \mathbb{R}^n$ *("region of attraction of the state feedback NMPC") with* $0 \in \mathcal{R}$ *such that:*

1. *The stage cost* $F : \mathcal{R} \times \mathcal{U} \to \mathbb{R}$ *is continuous, satisfies* $F(0,0) = 0$*, and is lower bounded by a class* $\mathcal{K}$ *function*[2] α_F*:* $\alpha_F(\|x\| + \|u\|) \leq F(x,u) \quad \forall (x,u) \in \mathcal{R} \times \mathcal{U}$.
2. *The optimal control* $\bar{u}^\star(\tau; x)$ *is piecewise continuous and locally Lipschitz in* x *in* $\mathcal{R}$*, uniformly in* τ*. That is, for a given compact set* $\Omega \subseteq \mathcal{R}$ $\|\bar{u}^\star(\tau; x_1) - \bar{u}^\star(\tau; x_2)\| \leq L_u \|x_1 - x_2\| \;\; \forall \tau \in [0, T_p),\; x_1, x_2 \in \Omega$*, where* L_u *denotes the Lipschitz constant of* $\bar{u}^\star(\tau; x)$ *(as a function of* x*) in* Ω.
3. *The value function, which is defined as the optimal value of the cost for every* $x \in \mathcal{R}$ $V(x) := J(\bar{u}^\star(\cdot; x); x)$ *is Lipschitz for all compact subsets of* $\mathcal{R}$ *and* $V(0) = 0$*,* $V(x) > 0$ *for all* $x \in \mathcal{R}/\{0\}$.
4. *Along solution trajectories starting at a sampling instant* t_i *at* $x(t_i) \in \mathcal{R}$*, the value function satisfies*

$$V(x(t_i + \tau)) - V(x(t_i)) \leq -\int_{t_i}^{t_i+\tau} F(x(s), u(s)) ds, \;\; 0 \leq \tau.$$

To establish the output feedback stability result it is furthermore necessary that for any compact subset $\mathcal{S} \subset \mathcal{R}$ we can find a compact outer approximation $\Omega_c(S)$ that contains $\mathcal{S}$ and is invariant under the NMPC state feedback.

Assumption 3 *For all compact sets* $\mathcal{S} \subset \mathcal{R}$ *there is at least one compact set* $\Omega_c(\mathcal{S}) = \{x \in \mathcal{R} | V(x) \leq c\}$ *such that* $\mathcal{S} \subset \Omega_c(\mathcal{S})$.

Assumptions 2.1 and 2.4 are satisfied by many stabilizing NMPC schemes. In principle Assumptions 2.2, 2.3 and 3 can also be satisfied. However, in general it is difficult to check them a priori, see [8–10].

[1] Note that similar to[8] we can in principle also allow $\mathcal{H}$ to depend on input derivatives. However, this would complicate the presentation and is thus not considered here.

[2] A continuous function $\alpha : [0, \infty) \to [0, \infty)$ is a $\mathcal{K}$ function, if it is strictly increasing and $\alpha(0) = 0$.

2.2 High Gain State Estimation

The system state is recovered by an high-gain observer. Application of the coordinate transformation $\zeta := \mathcal{H}(x,u)$, where $\mathcal{H}$ is the observability mapping, to the system (1) leads to the system in observability normal form in ζ coordinates

$$\begin{aligned}\dot{\zeta} &= A\zeta + B\phi(\zeta,u),\\ y &= C\zeta.\end{aligned}$$

The matrices A, B and C have the following structure

$$\begin{aligned}A &= \text{blockdiag}\,[A_1,\dots A_p]\,,\; A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & & & & \vdots\\ 0 & \cdots & \cdots & 0 & 1\\ 0 & \cdots & \cdots & \cdots & 0\end{bmatrix}_{r_i\times r_i}\\ B &= \text{blockdiag}\,[B_1,\dots,B_p]\,,B_i = \begin{bmatrix}0 & \cdots & 0 & 1\end{bmatrix}^T_{r_i\times 1}\\ C &= \text{blockdiag}\,[C_1,\dots,C_p]\,,C_i = \begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}_{1\times r_i},\end{aligned}$$

and $\phi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ is the "system nonlinearity" in observability normal form. The high-gain observer[3]

$$\dot{\hat{\zeta}} = A\hat{\zeta} + H_\epsilon(y - C\hat{\zeta}) + B\hat{\phi}(\hat{\zeta},u) \tag{4}$$

allows recovery of the states [2, 22] ζ from information of $y(t)$ assuming that

Assumption 4 *$\hat{\phi}$ in (4) is globally bounded.*

The function $\hat{\phi}$ is the approximation of ϕ that is used in the observer. The observer gain matrix H_ϵ is given by $H_\epsilon = \text{blockdiag}\,[H_{\epsilon,1},\dots,H_{\epsilon,p}]$, with $H^T_{\epsilon,i} = [\alpha_1^{(i)}/\epsilon,\; \alpha_2^{(i)}/\epsilon^2,\dots,\alpha_n^{(i)}/\epsilon^{r_i}]$, where ϵ is the so-called high-gain parameter since $1/\epsilon$ goes to infinity for $\epsilon \to 0$. The $\alpha_j^{(i)}$s are design parameters and must be chosen such that the polynomials

$$s^n + \alpha_1^{(i)} s^{n-1} + \cdots + \alpha_{n-1}^{(i)} s + \alpha_n^{(i)} = 0,\; i = 1,\dots,p$$

are Hurwitz.

Note that estimates obtained in ζ coordinates can be transformed back to the x coordinates by $\hat{x} = \mathcal{H}^{-1}(\zeta,u)$.

2.3 Semi-Global Practical Stability

The overall output feedback control presented in [7, 8] consists of the NMPC state feedback controller and a high-gain observer. The open-loop input is only calculated at the sampling instants using the state estimates of the observer. The observer itself operates continuously. Assuming that $\hat{x}(t_i) \in \mathcal{R}$, the input applied to the system is given by:

$$u(t) := \bar{u}^*(t - t_i; \hat{x}(t_i))$$

where $\bar{u}^*(\cdot;\hat{x}(t_i))$ is the optimal open-loop input signal of the NMPC optimal control problem (2) obtained at time t_i using the state estimate $\hat{x}(t_i)$ for prediction. The estimated state $\hat{x}(t_i)$ is given by

$$\hat{x}(t_i) = \mathcal{H}^{-1}(\hat{\zeta}(t_i), u(t_i;\hat{x}(t_{i-1}))),$$

where $\hat{\zeta}(t_i)$ is the high-gain observer state in observability normal form. Thus, in between sampling instants t_i to t_{i+1} an open-loop input is applied to the system.

Note that the observer estimate is not bounded to the feasibility region $\mathcal{R}$ of the NMPC controller. Since the open-loop optimal control problem will not have a solution outside $\mathcal{R}$, we define the input in this case to an arbitrary, bounded value.

The results in [7, 8] are derived in scaled observer coordinates. For this we define η as the scaled observer error,

$$\eta = [\,\eta_{11},\dots,\eta_{1r_1},\dots,\eta_{p1},\dots,\eta_{pr_p}\,],\;\text{ with }\;\eta_{ij} = \frac{\zeta_{ij} - \hat{\zeta}_{ij}}{\epsilon^{r_i - j}}.$$

Hence $\hat{\zeta} = \zeta - D_\epsilon\eta$ with $D_\epsilon = \text{blockdiag}[D_{\epsilon,1}, D_{\epsilon,2},\dots,D_{\epsilon,p}]$, $D_{\epsilon,i} = \text{diag}\left[\epsilon^{r_i-1},\dots,1\right]$. Under the stated assumptions using the scaled observer error, it is shown in [8] that the output feedback scheme can achieve practical stability: For any small set containing the origin, there exists an observer gain and a sampling time such that the trajectories converge to the set in finite time and stay inside the set.

Theorem 1 (Semi-global practical stability)
Given arbitrary compact sets $\mathcal{Q} \subset \mathbb{R}^n$ and $\mathcal{S} \subset \mathcal{R}$. Then, for any $\rho > 0$, there exists $\delta_1^\star > 0$ such that for $0 < \delta < \delta_1^\star$, there exists an $\epsilon_1^\star > 0$ such that for all $0 < \epsilon < \epsilon_1^\star$, and all $(x_0,\eta_0) \in \mathcal{S} \times \mathcal{Q}$, the trajectories $(x(t),\eta(t))$ stay bounded, converge in finite time to the set $\|(x,\eta)\| \le \rho$, and $x(t) \in \mathcal{R}$ $\forall t \ge 0$.

Note that the high-gain observer is formulated in scaled observer coordinates, while the NMPC controller is formulated in the original coordinates.

3 Output-Feedback NMPC using High-Gain Observers in Original Coordinates

The output feedback NMPC approach outlined in the previous section uses a high-gain observer in observability normal form coordinates, and relies on explicit knowledge of $\mathcal{H}^{-1}$. Often, however, it is difficult to obtain $\mathcal{H}^{-1}$ explicitly. In this section we first outline an observer where the explicit knowledge $\mathcal{H}^{-1}$ can be avoided to a large extent, and then present an observer where the knowledge is avoided alltogether. The latter approach allows to design the NMPC state feedback controller and the state estimator directly in original coordinates and simplifies the over all design.

One simple way to avoid the explicit knowledge of $\mathcal{H}^{-1}$ to a large extent is to set $\hat{\phi} = 0$ in (4), which satisfies Assumption 4. Then the observer is given by

$$\dot{\hat{\zeta}} = A\hat{\zeta} + H_\epsilon(y - C\hat{\zeta}).$$

[3] We use hatted variables for the observer states and variables.

In this case the observer error still converges to any desired bound for any sufficiently small ϵ [20, 21]. However, the performance might degrade significantly. The key advantage is that the inverse of the observability map $\mathcal{H}$ is only necessary at the sampling instant. Rewriting equation (2.3) leads to

$$\hat{\zeta}(t_i) = \mathcal{H}(\hat{x}(t_i), u(t_i; \hat{x}(t_{i-1}))). \tag{5}$$

In principle this equation, together with the known values of $\hat{\zeta}(t_i)$ and $u(t_i, \hat{x}(t_{i-1}))$, can be added to the dynamic optimization problem (2) that is solved in the NMPC controller at time t_i. This does not change the solution of (2), since the value of $\hat{x}$ is, due to the uniform complete observability assumption, uniquely defined by (5).

Another possibility to avoid explicit information on $\mathcal{H}^{-1}$ is to rewrite the observer equations in terms of the original coordinates [4, 13]. It has the advantage that $\mathcal{H}^{-1}$ is not directly needed for the implementation of the observer. As a starting point consider the high-gain observer in the ζ coordinates as given in (4), assuming $\phi = \hat{\phi}$ (no mismatch between the estimator and the real system)

$$\dot{\hat{\zeta}} = A\hat{\zeta} + B\phi(\hat{\zeta}, u) + H_\epsilon(y - \hat{y}). \tag{6}$$

Applying $\hat{x} = \mathcal{H}^{-1}(\hat{\zeta}, u)$, which exists due to Assumption 1 using

$$\dot{\hat{\zeta}} = \frac{\partial \mathcal{H}}{\partial \hat{x}} \dot{\hat{x}} \tag{7}$$

we obtain:

$$\dot{\hat{x}} = \left[\frac{\partial \mathcal{H}}{\partial \hat{x}}(\hat{x}, u)\right]^{-1} (A\mathcal{H}(\hat{x}, u) + B\phi(\mathcal{H}(\hat{x}, u), u)) + \left[\frac{\partial \mathcal{H}}{\partial \hat{x}}(\hat{x}, u)\right]^{-1} H_\epsilon(y - \hat{y}).$$

The first part is equivalent to $f(x, u)$ which leads to:

$$\dot{\hat{x}} = f(\hat{x}, u) + \left[\frac{\partial \mathcal{H}}{\partial \hat{x}}(\hat{x}, u)\right]^{-1} H_\epsilon(y - h(\hat{x}, u)). \tag{8}$$

Thus, it is "only" necessary to know $\left[\frac{\partial \mathcal{H}}{\partial x}(\hat{x}, u)\right]^{-1}$. If $\left[\partial \mathcal{H}/\partial x(\hat{x}, u)\right]^{-1}$ is not known, one can left-multiply (8) by $\partial \mathcal{H}/\partial x$. The resulting system can be efficiently solved using DAE integrators. Note that, similarly to $\mathcal{H}$, it is normally not a problem to obtain $\frac{\partial \mathcal{H}}{\partial x}$ from the system equations (1) via simple calculations or using automatic algebraic differentiation.

The same observer structure is used in [4, 13]. In [13] an additional projection is required since systems that are not uniformly completely observable and control laws that are not globally bounded are considered. This projection is not necessary here, since the input resulting from the NMPC controller is bounded, and since we limit ourself to uniformly completely observable systems.

Since the convergence properties of the observer (8) is equivalent to the observer in observability coordinates, the stability results given in Section 2 stay unchanged. The approaches outlined allow to implement the high-gain observer and NMPC controller in original coordinates, leading to a simplified implementation and design.

4 Example

In this section the output feedback NMPC approach using the high-gain observer in original coordinates is applied to the control of a continuous mixed culture bioreactor as presented in [11]. The schematics of the considered process is shown in Figure 1. The system consists of a culture of two cell strains

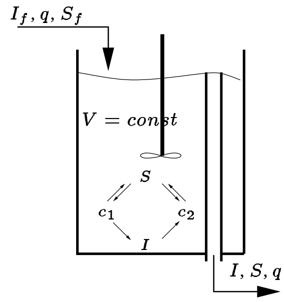

Figure 1: Continuous mixed culture bioreactor.

that have different sensitivity to an external growth-inhibiting agent. The cell density of the inhibitor resistant strain is denoted by c_1, the cell density of the inhibitor sensitive strain is denoted by c_2, and the substrate and inhibitor concentrations in the reactor are denoted by S and I. Simple material balances lead to:

$$\begin{aligned}
\frac{dc_1}{dt} &= (\mu_1(S) - D)\, c_1, \\
\frac{dc_2}{dt} &= (\mu_2(S, I) - D)\, c_2, \\
\frac{dI}{dt} &= -pc_1 I + D\,(I_f - I)\,, \\
\frac{dS}{dt} &= -\mu_1(S)\frac{c_1}{Y_1} - \mu_2(S, I)\frac{c_2}{Y_2} + D\,(S_f - S)\,.
\end{aligned}$$

Here D is the dilution rate q/V and Y_1, Y_2 are the yields of the species and $\mu_1(S)$ and $\mu_2(S, I)$ are the growth rates of the species, which are described by Monod type expressions. The inputs to the system are the dilution rate $u_1 = D$ and the inhibitor addition rate $u_2 = DI_f$. The outputs available for control are $y_1 = \ln \frac{c_1}{c_2}$, which can be thought of as an turbidity measurement and the cell density of species one, $y_2 = c_1$. The control objective is to stabilize a given steady state. Note, that the system satisfies the assumptions required for applying the output feedback scheme proposed, i.e. the observability map given by $\mathcal{H} = [y_1, \dot{y}_1, \ddot{y}_1, y_2]$ is (locally) invertible in the region of interrest. We use the high-gain observer outlined in Section 3 for state recovery, where the observer is implemented using a DAE integrator to "avoid" the explicit inversion of $\partial \mathcal{H}/\partial x(\hat{x}, u)$. As state feedback NMPC scheme, the quasi-infinite horizon NMPC strategy [3] with a (sufficiently small) fixed sampling time of $\delta = 2$hrs and a prediction horizon $T_p = 20$hrs is used. The optimal input at every sampling instant is obtained by mathematical programming parameterizing the input as piecewise constant, with 10 control intervals. Thus the derivative of the input over one sampling time is zero, i.e. once continuously differentiable. The cost F weighs the quadratic deviation of the states and inputs from their steady state values. The parameters α for $\mathcal{H}_\epsilon$ are chosen to: $\alpha_1^{(1)} = 2$, $\alpha_2^{(1)} = \sqrt{2}$, $\alpha_3^{(1)} = 1$ and $\alpha_1^{(2)} = 1$. To illustrate the stability and performance of the closed loop we consider different observer gains ϵ while keeping (the sufficiently small) sampling time δ constant. In all simulations the observer is initialized with the correct values for c_1 and c_2 (since they can be directly obtained from the measurements), whereas the other observer states are initialized with the steady state value. Figure 2 exemplary shows closed loop system trajectories projected onto the c_1/c_2 phase plane for different observer gains $k = \frac{1}{\epsilon}$ in comparison to the state feedback NMPC controller starting from the same initial condition. Figure 3 shows the corresponding

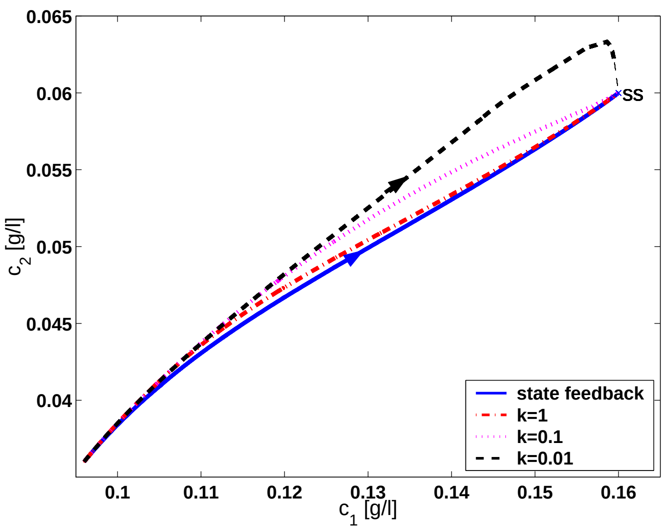

Figure 2: Phase plot of c_1 and c_2

time behavior of the inhibitor concentration I and the inhibitor addition rate (input u_2) for different values of ϵ. Additionally, the real cost occurring, i.e. the integrated quadratic error between the steady state values for the states and inputs in transformed coordinates, is plotted. One can clearly see the in this example utilized over the sampling time constant input to the system. Furthermore, as can be seen, the larger the observer gain (the smaller ϵ), the closer the trajectories converge to the state feedback case. The cost of the output feedback controller approaches the cost of the state feedback controller for lower ϵ, which shows the recovery of performance. Notice that we use relatively low gains for the observer, meaning that ϵ is large. This example verifies the stability of the closed loop and the recovery of performance for increasing values of the observer gain using a high-gain observer in original coordinates.

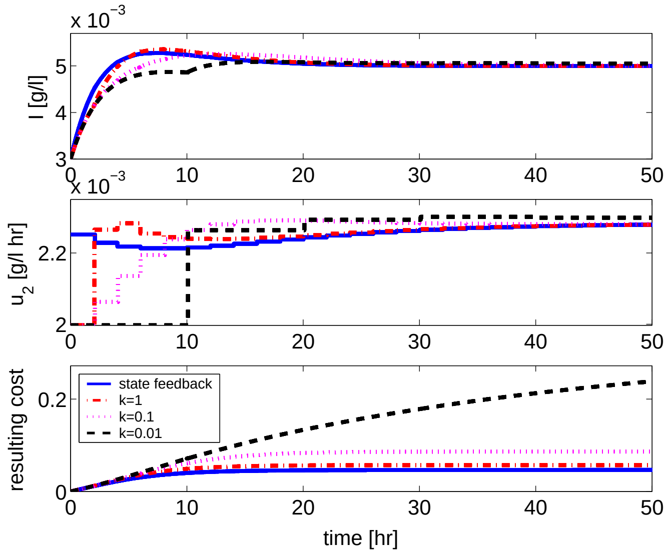

Figure 3: Trajectories of I, u_2 and summed up cost

5 Conclusions

To derive stable output feedback NMPC is of practical as well as of theoretical relevance. In this paper we showed that the sampled-data output feedback NMPC approach for continuous time systems derived in [7, 8] also allows to use high-gain observer in original coordinates. This is, as shown, advantageous, since it avoids the explicit knowledge of the inverse of the observability map. In the resulting scheme both the observer and the NMPC controller are formulated in original coordinates. The high-gain observer used involves the (local) inverse of the Jacobian of the observability map with respect to the state. As outlined, using DAE integrators it is even possible to avoid the explicit knowledge of this inverse, making the implementation of the high-gain observer rather simple. The resulting closed-loop keeps the same stability properties as the approach given in [7, 8], namely semi-global practical stability.

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