Dantzig-Wolfe decomposition for real-time optimization - applied to the Troll west oil rim

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Abstract: This paper studies different decomposition approaches for real-time optimization of process systems with a decentralized structure where the idea is to improve computational efficiency and transparency of a solution. The contribution lies in the application and assessment of the Dantzig-Wolfe method which allows us to efficiently decompose a real-time optimization problem into parts. Furthermore, the nonlinear system is modeled by piecewise linear models with the added benefit that error bounds on the solution can be computed.
The merits of the method are studied by applying it to a semi-realistic model of the Troll west oil rim, a petroleum asset with severe production optimization challenges due to rate dependent gas-coning wells. This study indicates that the Dantzig-Wolfe approach offers an interesting and robust option for complex production systems. Moreover, the method compares favourable with earlier results using Lagrangian relaxation which again was favourable compared to a global approach.

Keywords: Optimization, Dantzig-Wolfe Decomposition, Petroleum production, Plantwide control.

1. INTRODUCTION

Development of a petroleum field asset requires planning on multiple horizons. On a life-cycle horizon, strategic decisions are made on field development such as the choice of technology and export options, and investment and recovery strategies. For offshore assets the choice of technology may include subsea solutions, and the issue of processing the reservoir fluid offshore or onshore (Nygreen et al., 1998). On a medium time horizon, typically three months to two years, production targets are decided. Depending on the life cycle of an asset, decisions may also involve a drilling program. During for instance the green field stage, it is important to plan, drill and commision new wells to reach some pre-defined plateau rate as soon as possible. A reservoir simulator, containing anything between 100.000 and $1,000,000$ states, is usually an important planning tool on the medium time horizon. A reservoir simulator will be quite complex if the geology is complex, due to heteregeneities like faults and shale layers, to represent flow patterns accurately.

On a shorter time horizon, typically days to weeks, production optimization where both the sub-surface part, like the reservoir and wells, *and* the surface part like the manifolds, pipelines and downstream production equipment, is taken into account is important. This is commonly called the real-time production optimization (RTPO) problem. Production may be constrained by reservoir conditions such as coning effects and/or the production equipment like pipeline capacity or downstream water handling capacity, and constraints may move from one part of the system to another part over time. Water production may e.g. be low early on and increase dramatically during the decline phase of a reservoir thereby making water handling capacity an issue. Decision variables in RTPO include production and possibly injection rates, and routing of well streams. A typical production system structure is shown in Fig.1. It has two separate non-connected reservoirs from which 11 wells feed into three manifolds and pipelines, and finally into the downstream facilities section. Manifold 1 and 2 belong to one cluster, while manifold 3 belongs to the other cluster.

RTPO is in use in the upstream industry today. Wang (2003), Saputelli et al. (2003) and Bieker et al. (2006) provide readable overviews. It might be noted that these references focus on the value chain from the reservoir to, and not including, the downstream processing equipment. The downstream boundary is typically a constant pressure on the inlet separator. A few publications on RTPO for the production chain from the reservoir to export are available; Foss and Halvorsen (2009), Selot et al. (2007). Commercial products for RTPO are available, but not widely used. Two of them are GAP and MaxPro. They model the well and near well region, and the pipeline system, and solve the optimization problem using a nonlinear programming (NP) algorithm like sequential quadratic programming (SQP).

There are several factors which complicate the RTPO problem.

- RTPO may give rise to optimization problems with both continuous and discrete decision variables. Discrete decision variables are found in routing when there is a choice to route the fluid from a manifold to one of several flow lines. The presence of discrete decision variables complicates the optimization problem by transforming a linear program (LP) or a nonlinear program (NP) to a mixed integer linear program (MILP) or a mixed integer nonlinear program (MINLP), respectively. Güyaguler and Byer (2007) discusses RTPO in the context of MILP while Kosmidis et al. (2005) use a MINLP formulation.
- The models in the optimization problem are often nonlinear, some of which may be highly nonlinear, as will be discussed later. This includes well models as well as pressure drop models for the pipelines which support multiphase fluid transport.
- The optimization problems are usually quite large and may include several hundred decision variables. An example is the rate allocation problem at Troll which in total includes more than one hundred wells (Hauge and Horn, 2005).

This paper focusses on the RTPO problem for systems with a decentralized structure meaning that common constraints are quite few. Such strucures are quite typical in the upstream petroleum industries as visualized by Fig.1. The contribution lies in the application and assessment of the Dantzig-Wolfe method which allows us to decompose a RTPO problem into parts meaning that we apply a divide-and-conquer strategy which is a sound engineering design principle. This principle has survived ever since complex systems came into making. The Dantzig and Wolfe principle dates back to 1960 (Dantzig and Wolfe, 1960).

A few recent publications apply Dantzig-Wolfe decomposition (DWD) to process systems. Alabi and Castro (2009) apply DWD to a refinery planning problem, formulated as a large LP problem, by decomposing it along the value chain. They show substantial savings in computation time. They also point to the inclusion of binary decision variables, which will be addressed in this paper, as a future task. Cheng et al. (2008) propose DWD as a means for designing a decentralized MPC for plant-wide MPC coordination. Again substantial computational savings are reported for the LP formulation chosen. This paper also gives a very readable introduction to DWD. Both papers state that DWD is particularly well suited for large problems with well-structured subproblems and a small number of linking constraints.

This study will show that the Dantzig-Wolfe approach offers an interesting and robust option for complex production systems wiht certain structural properties. Moreover, the method compares favourable with earlier results using Lagrange relaxation (LR) (Foss et al., 2009) on a realistic field case. The nonlinear system will be modeled by piecewise linear models with the added benefit that error bounds on the solution of the production optimization problem can be computed.

The remainder of this paper is organized as follows. First, the RTPO problem is presented in a mathematical context before the decomposition approach in general and the Dantzig-Wolfe method in particular are presented.

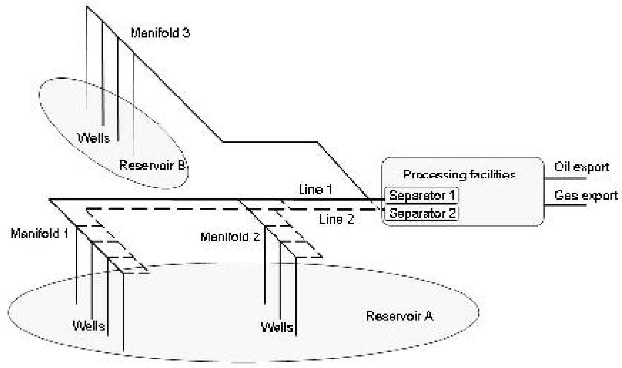

Fig. 1. A petroleum production system with two separate reservoirs from which 11 wells feed into 3 manifold and 3 pipelines. Manifold 1 and 2 belong to one cluster, while manifold 3 belongs to the other cluster. The pipeline flows provide input to the processing facilities where fluids are conditioned for export.

Subsequently, the Troll west oil rim case is presented and results are shown. Finally, results are discussed and some conclusions end the paper.

2. FORMALIZING THE RTPO PROBLEM

The RTPO problem will in most cases mean maximizing oil production while honouring system constraints like capacities in pipelines and wells, safety regulations and preventing damage on long-term effects, in particular recovery of available hydrocarbon resources. The latter point is important. An example of the interplay between short term production and long-term recovery was shown in Naus et al. (2006) in the sense that accelerated short-term production reduced long-term recovery.

The optimization problem is usually treated in a quasi-dynamic way by re-optimizing a stationary optimization problem, typically once a day. The solution of the mathematical RTPO will serve as a recommendation to the operating engineers who may or may not follow the advise. One reason for neglecting a recommendation may hinge on the fact that the transition cost of changing from one routing configuration to another is not included in the optimization problem. Therefore such a change will only be implemented if there is a substantial gain by doing this.

Referring again to Fig.1 to explain upstream systems closer, there are four wells connected to manifold 1 and 3, respectively, and three wells connected to manifold 2. Well streams from each well are connected to one pipeline. Hence, each of the wells in manifold 1 and 2 can be connected to either of the two pipelines transporting the reservoir streams to downstream processing. There is only one pipeline from manifold 3, and therefore no routing decision is necessary in this part of the system. The decision variables on each well are usually one production choke valve to adjust production and on-off valves linking a well to one of the pipelines. Further, different well completions may give rise to additional decision variables like the injected gas-rate for a gas-lift well, a commonly used technology to increase well lifetime as reservoir pressure decreases. Altogether this means that there will be both continuous as well as discrete decision variables in a typical

RTPO problem. The well and pipeline system is divided into clusters. There are two cluster in Fig.1, one covering manifold 1 and 2, and the second includes manifold 3. Hence, a cluster may include one or more manifolds.

We focus on production systems with a decentralized structure where common constraints may include downstream processing capacity limitations and common pipelines. In the following we present a system model which encompasses a large class of upstream production systems. Some simplifications are made to ease the explanation. For instance we assume only one manifold for each cluster in this section. An extension to several manifolds per cluster, as is the case for manifold 1 and 2 in Fig.1, is however straightforward. In fact in the Troll case treated later the clusters have two manifolds each.

We first present a system model for a subsystem, denoted cluster i, before the integrated optimization problem is described.

Indexes, constants and decision variables are explained in Table 1 and 2.

2.1 Modelling a subsystem

In the following we present and comment the model of *one* subsystem, cluster i.

- Mass balance is preserved for each phase, i.e. gas, oil and water, at each node. This means that no phase transition takes place at the surface of a cluster.

$$\sum_{j=1}^{J(i)} q_{ij}^p = q_i^p, \quad p \in \{g, o, w\} \tag{1}$$

- The routing problem is parameterized through binary variables for each well, one for each line; y_{ij}^l. If $y_{ij}^l = 1$ the well is connected to line l, if not it is zero. Each well cannot be connected to more than one line, hence

$$\sum_{l=1}^{L(i)} y_{ij}^l \leq 1, \quad y_{ij}^l \in \{0, 1\}, \quad j \in \{1, \ldots, J(i)\} \tag{2}$$

  This implies that the flow q_i^p from one cluster is divided onto $L(i)$ pipelines.
- The well model, or performance curve for gas, oil and water, are given by the following nonlinear structure

$$q_{ij}^p = d_{ij}^p(p_{ij}^{res}, p_{ij}^{wh}), \tag{3}$$
$$p \in \{g, o, w\}, \quad j \in \{1, \ldots, J(i)\}$$

  where p_{ij}^{res} and p_{ij}^{wh} denotes the reservoir pressure locally at the well and the pressure at the wellhead, respectively. Depending on the reservoir conditions near a well the complexity of these well models vary a lot. The simplest version will be a linear model. In systems with rate-dependent gas coning however, as in the Troll oil case (Hauge and Horn, 2005), nonlinearities can be severe.
- The pressure drop across the production choke is given by

$$y_{ij}^l p_i^l \leq p_{ij}^{wh}, \tag{4}$$
$$j \in \{1, \ldots, J(i)\}, \quad l \in \{1, \ldots, L(i)\}$$

  This constraint may only be binding if $y_{ij}^l = 1$ since it is always satisfied for $y_{ij}^l = 0$.
- Flow into the pipelines from cluster i to the platform is given by

$$q_i^{pl} = \sum_{j=1}^{J(i)} y_{ij}^l q_{ij}^p \tag{5}$$
$$p \in \{g, o, w\}, \quad l \in \{1, \ldots, L(i)\}$$

- The pressure drop in a pipeline segment from cluster i to the inlet separator depends nonlinearily on the flow of gas, oil and water in the pipe segment. The nonlinearities are particularly severe during the transition from one multiphase flow regime to another, and when a pipeline exhibits slugging. More on multiphase flow may e.g. be found in Brenne (2005)

$$p^{sep} - p_i^l = d_i^l(q_i^{gl}, q_i^{ol}, q_i^{wl}), \tag{6}$$
$$l \in \{1, \ldots, L(i)\}$$

- There are non-negativity conditions on all flow and pressure variables, i.e. backflow is not modeled.

It should be added that the downstream boundary condition is given by a fixed inlet separator pressure p^{sep}, and we assume that it is equal for all $L(i)$ pipelines. Further, it is straightforward to include additional local constraints like for instance the flowrate from a well due some external reason. This could be well-related problems like sand production, or reservoir based constraints as discussed earlier. Such contraints will typically induce relations like $q_{ij}^o + q_{ij}^w \leq q_{ij}^{\max}$

Table 1. The indexes used.

i	-	cluster i
I	-	no. of clusters
$p \in \{g, o, w\}$	-	phase index - gas, oil or water
ij	-	well j in cluster i
$J(i)$	-	no. of wells linked to cluster i
$l \in \{1, .., L(i)\}$	-	line index for cluster i
$L(i)$	-	no. of lines linked to cluster i

Table 2. The variables and data used to define the sub-problem.

q_i^p	-	total mass flowrate of phase p from cluster i
q_{ij}^p	-	mass flowrate of phase p from well j in cluster i
q_i^{pl}	-	mass flowrate of phase p through line l in cluster i
y_{ij}^l	-	binary variable equal to 1 if well ij is routed to line l
p_{ij}^{res}	-	reservoir pressure at well ij
p_{ij}^{wh}	-	wellhead pressure
d_{ij}^p	-	well performance model
d_i^l	-	pipeline pressure drop model
p_i^l	-	pressure in line l subsea in cluster i
p^{sep}	-	separator pressure

2.2 The integrated problem

The RTPO problem is specified below. The objective function is defined by the total oil production, and the global constraints are given by gas and water handling capacities in the downstream part of the value chain. Hence, the objective function and common constraints are given by

$$max \sum_{i=1}^{I} q_i^o \tag{7}$$

$$\sum_{i=1}^{I} q_i^g \leq \overline{q}^g \tag{8}$$

$$\sum_{i=1}^{I} q_i^w \leq \overline{q}^w \tag{9}$$

The objective function and common constraints are linear and additive and each term $q_i^o = \sum_{j=1}^{J(i)} q_{ij}^o$ is a function of only local variables.

The complete RTPO problem consists of I clusters, each modelled by (1)-(6), or an extension of these equations due to several manifolds in one cluster, and the integration through (7)-(9).

The actual decision variables are production choke openings and on-off valves linking a well to a pipeline. The production choke openings are not directly a part of the optimization problem. They are calculated using the pressure drop across the production choke (4) and the flowrate through the production choke $(q_{ij}^g, q_{ij}^o, q_{ij}^w)$ in an appropriate valve model.

2.3 Piecewise linearization and SOS2 sets

The optimization problem contains both continuous and discrete variables. Furthermore, nonlinear well and pressure drop models are present. Hence, this is basically a MINLP problem. We transform this into a MILP problem by replacing the nonlinear constraints by linear constraints and constraints on some auxiliary integer variables. The procedure is as follows: The nonlinear constraints, (3) and (6), are replaced by piecewise linear approximations. These piecewise linear approximations are modelled by linear constraints and discrete variables, i.e. integer constraints, using Special Ordered Sets of type 2 (SOS2). The discrete variables are necessary to assure interpolation between neighbouring points only, Williams (2005), as in any piecewise linear approximation of a nonlinear function. The number of linear constraints and integer constraints necessary to replace one nonlinear constraint depends on the nonlinearities and approximation accuracy. Higher accuracy means more interpolation points and hence more linear and integer constraints.

3. DECOMPOSITION

3.1 Principle

When a problem becomes too large or complicated to handle, a decomposition approach can be applied if the problem structure is suitable. The basic mechanism in all decomposition principles is to decompose the original problem into smaller sub-problems which are coordinated by a "master" problem. There exists multiple decomposition techniques to solve large problems. Two common methods are Lagrange relaxation and Dantzig-Wolfe decomposition.

Both LR and DWD are suited for problems with a block angular constraint structure which is the case for the RTPO problem described above. The structure is exploited when the original problem is split into sub-problems, while the common constraints remain in the master problem.

In LR (Beasley, 1993) the basic idea is to attach Lagrange multipliers to the common constraints in the model and relax these in the objective function, while DWD handles the common constraints in a master problem. The resulting integrated optimization problem will hence fall apart into I local optimization problems, one for each cluster i (Fisher, 1985). For (convex) LP problem the solution of I such local optimization problems provides the same solution as (1)-(9) provided that the Lagrange multipliers for the common constraints λ^g, λ^w are known. Hence, the Lagrange multipliers put a common cost to the use of a scarce resource by each local problem.

3.2 Dantzig-Wolfe decomposition

When applying DWD to the RTPO problem the subproblems will be identical to LR. However, while the Lagrange multipliers are updated by a simple heuristic in the LR case, the update is now done by solving an LP-problem.

We start by assuming linear constraints and continous variables, i.e. an LP-problem instead of a MILP problem. The master problem is a reformulation of the integrated problem. By taking advantage of the fact that a convex combination of basic feasible points, which are corner points of the feasible set defined by the linear constraints of the integrated problem, also is a feasible solution, an alternative formulation can be achieved. Each basic feasible point in the integrated problem is then represented as a variable in the master problem. The number of basic feasible points for any practical problem can clearly be prohibitively high, and in reality only a small number of these basic feasible points will ever enter the basis in the master problem. The idea is then to restrict the master problem by reducing the number of basic feasible points. This is called a Restricted Master Problem (RMP).

Hence, we start with a few basic feasible points and check if the solution of the integrated problem is within a convex combination of these points. If this is not the case new basic feasible points are included in a structured way until the optimal solution has been found (Williams, 2005). This is usually called column generation and several procedures are proposed in the literature; either adding one or several columns, i.e. new basic feasible points, at each iteration (Dantzig and Thapa, 2002). Some details of the algorithm are given below with some related comments specific to the RTPO problem.

Algorithm structure

1. Choose two initial basic feasible points for each local optimization problem.

2. Specify the RMP as a LP for the given set of basic feasible points. Then solve it and compute values for the Lagrange multipliers for the global constraints, i.e. λ^g, λ^w. The RMP is specified in a separate section below.

3. Solve I local optimization problems by using the Lagrange multipliers computed in 2.

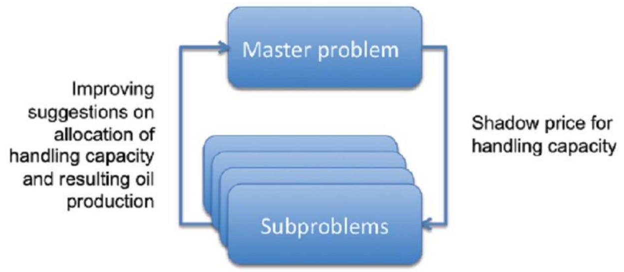

Fig. 2. Iteration structure for Dantzig-Wolfe Decomposition (DWD) and Lagrangian Relaxation (LR)

4a. For $i \in \{1,\ldots,I\}$: If the solution of a local optimization problem i extends the convex set defined by the basic feasible points used in 2, then add these basic feasible points to the RMP, and go to 2. (This implies that the feasible region of this new RMP is extended).

4b. If the solutions of all the local optimization problems are unchanged, the optimal solution has been found; and the algorithm terminates.

The main iteration loop is shown in Fig.2. This figure is also applicable for LR if the master problem box is understood as the updating algorithm for the the Lagrange multipliers for the global constraints.

In view of our RTPO problem item 1 above implies that two feasible solutions for each cluster must be determined to start the algorithm.

3.3 Restricted master- and sub-problem

The procedure is to update the Lagrange multiplier in a way that the consumption of the relaxed common constraints converge to their optimal values. Each subproblem is defined by (again only including two common constraints)

$$\begin{aligned} max \quad & q_i^o - \lambda^g q_i^g - \lambda^w q_i^w - \lambda_i^{CONVEX} \\ & \lambda \geq 0, \quad i \in \{1,\ldots,I\} \end{aligned} \tag{10}$$

and the local constraints (1)-(6). λ_i^{CONVEX} is the Lagrange multiplier for the convexity constraint in the RMP defined below. Since no sub-problem variables are associated with it, it will only act as a constant in the subproblem.

The RMP can now be formulated. z_{is}^p represents one basic feasible point s from sub-problem i. z_{is}^p could in principle include the optimal value of all decision variables for subproblem i after solving it given λ^g and λ^w. However, only the variables also present in the objective function and the common constraints will be relevant for the RMP. Hence, z_{is}^p will for this RTPO problem contain some flow variables (q_i^o, q_i^g, q_i^w), but no pressure variables. μ_{is} is the corresponding weight the master problem will give this basic feasible point. The objective function of the master problem is given in (11). Further, (12) and (13) represents the constrained common resources, while (14) is the convexity constraint.

$$max \sum_i \sum_{s\in S_i} z_{is}^o \mu_{is} \tag{11}$$

$$\sum_i \sum_{s\in S_i} z_{isp}^g \mu_{is} \leq \overline{q}^g \tag{12}$$

$$\sum_i \sum_{s\in S_i} z_{isp}^w \mu_{is} \leq \overline{q}^w \tag{13}$$

$$\sum_{s\in S_i} \mu_{is} = 1 \quad i \in \{1,\ldots,I\} \tag{14}$$

$$\mu_{is} \geq 0 \tag{15}$$

3.4 Integer variables

DWD will find exact optimal solutions for feasible LP problems. If it is extended to a MILP problem, however, Branch & Price (Desrosiers and Lubbecke, 2006) or some heuristics have to be applied to handle the integer properties. When solving the master problem, we have not imposed integer restrictions on μ_{is}, i.e. the RMP is solved as an LP to achieve Lagrange multipliers for (12) - (14). The resulting solution may then be infeasible with respect to the original MILP problem, since a convex combination of two different basic feasible points is not necessarily feasible. As mentioned, this could be handled in several ways. However, if a satisfying number of basic feasible points are generated up front, a feasible solution could simply be found by demanding integer values for μ_{is} and solve the RMP as an MIP problem. Vanderbeck (2006) adresses the use of DWD on mixed integer problems.

3.5 Solution quality

For both LP and MILP problems, upper and lower bounds on the objective function can be computed. The LP solution of the RMP plus the sum of the objective values of the sub-problems will act as an upper bound (Karlof, 2006). In the LP case, the solution of the RMP alone will give a feasible lower bound, while for the MILP problems a heuristic has to be applied to create the feasible lower bound. By using these bounds actively during the optimization process, it is possible to terminate the optimization problem when an acceptable gap is achieved.

4. RESULTS

The Troll field is a huge oil and gas field on the continental shelf west of Norway. Production allocation is complex as described in (Hauge and Horn, 2005). We study the Troll C production system shown in Fig.3 where primarily oil is produced from an oil rim through more than 50 wells. Well models and pressure drop models for multiphase flow in pipelines are based on typical models as encountered in this application. Hence, the models should be understood as approximations of the actual well and pipeline models. Each nonlinear model is approximated by a piecewise linear model. A well model (3) is typically divided into somewhere between 10 and 100 linear segments with the wellhead pressure as its input. The pipeline models require more linear segments since they depend on three inputs, cf. (6). There are 8 clusters, and each cluster has a complex structure in the sense that they contain two manifolds. Each cluster has $6-8$ wells and the total number of wells is 64. For the moment only the gas handling capacity is a binding constraint. Water handling will become an issue

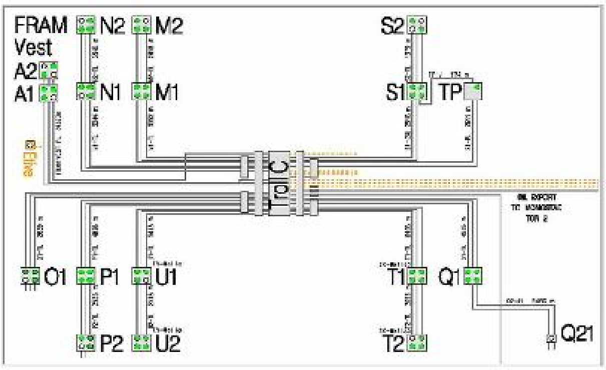

Fig. 3. Topology for the wells connected to the Troll C platform

in the near future as the reservoir drains and therefore produces more water.

The purpose of the numerical study is to investigate the DWD performance compared to a global strategy and the LR method proposed in Foss et al. (2009). Three different strategies were therefore defined:

(1) A global strategy where all clusters are solved in one large MILP problem.
(2) The LR method proposed in Foss et al. (2009).
(3) The DWD method proposed in this paper.

The computations were performed on an IBM Thinkpad T60P with a $2.33GHz$ processor and a tolerance bound of 0.5% for LR and DWD. The state-of-the-art XPress-MP software suite is used to solve the MILP problems. The main results are presented in Table 3. Results are presented column-wise for different system sizes starting with two cluster and ending with the full 64 well/8 cluster system. The gas capacity for each scenario increases with the number of clusters as shown in line 2. In the next three lines the number of variables and constraints are listed. Then follows the results in terms of computation time and oil production which is the ultimate goal, cf. (7).

Finally, it should be noted that the results in Table 3 represent typical values as observed after several test runs.

Table 3. Results from tests on the model of the Troll C production system.

No. of clusters	2	4	6	8
Gas cap.[Sm^3/day]	3000	12000	18000	24000
Continuous variables	13898	27805	41766	55688
Discrete variables	1029	2134	3725	4819
Constraints	491	981	1639	2185
Strategy 1 - Global				
Solution time [min]	0.26	7.38	237.0	720.0
Oil [Sm^3/day]	1777	6487	11641	14365
Strategy 2 - LR				
Solution time [min]	1.42	8.54	18.2	19.2
Oil [Sm^3/day]	1774	6467	11640	14440
Strategy 3 - DWD				
Solution time [min]	1.86	1.43	6.16	11.3
Oil [Sm^3/day]	1777	6458	11629	14473

5. DISCUSSION

The main observation to make is the fact that the decomposition stratgies, DWD and LR, outperform the global method for the combined rate allocation and routing problem for all but the smallest 2 cluster problem. The global method does not converge to its termination criteria after 12 hours for the eight cluster case and in general it has a hard time solving problems consisting of 6 clusters or more. Furthermore, DWD shows superior performance to LR.

By taking a closer look at the 2 cluster problem, the global method is actually fastest. This is not surprising as it is expected that the global method would be faster for small problems. For the medium size problem with 4 clusters, the global method is still working fine, and is actually faster than LR. DWD is in this case extremely quick, due to few main iterations. For the larger problems, we observe that the two decomposition methods are much faster than the global method, and that DWD is significantly faster than LR.

The reason why DWD is faster than LR is related to the updating of Lagrange multipliers. The DWD master problem finds good multipliers with fewer iterations than the LR master problem, and on average converges after fewer iterations. It should be mentioned that the computations involved in solving the DWD master problem, i.e. the LP-problem, is small compared to solving the local MILP-problems. Hence, this is no issue when comparing DWD and LD.

DWD is more stable with respect to solution time than LR. Furthermore, DWD has few tuning parameters, and works well for changing data sets. LR in contrast, is quite sensitive to perturbations of the data set. A minor change might result in a doubling of the solution time. However, extensive knowledge of the problem will give the operator a good feel for which parameter values result in fast convergence.

Focussing on solution quality, we observe that the global method finds the optimal solution for all except the full field problem with 8 clusters. In that case the method was stopped after 12 hours, with still a little more than 7.5% in duality gap. The decomposition methods terminate with less than 0.5% duality gap for all problems. The solution time does of course depend on the resolution of the piecewise linear models. A cruder approximation reduces run-time and vice versa.

DWD provides a framework for decomposing a problem and still keep track of the optimal solution for the integrated problem. This approach has potential advantages in terms of algorithmic efficiency as indicated by the test case in the previous section. The DWD algorithm, as well as LR, has some interesting properties. First, the sub-problems may be solved using different algorithms or even different software packages. This feature is interesting for integrated optimization applications which may encompass reservoir, wells, pipelines and processing facilities. It should be added that the duality gap can only be computed if upper and lower bounds on the solution can be found. This is in general not possible if the sub-problems are nonlinear programs as opposed to MILPs. A second useful property of the algorithmic structure is the potential for parallel computing since each sub-problem is self-contained and has no dependency on the other sub-problems. If the computational load between the sub-problems is well-balanced a parallel implementation will be particularly efficient.

The optimization problem is usually treated in a quasi-dynamic way by re-optimizing the stationary optimization problem, typically once a day. More frequent disturbances may be handled by selecting a couple of wells for frequent production changes to compensate variations in for instance gas processing capacity. Well models are typically updated twice a year by running well tests to collect data to estimate well parameters. The use of dynamic models is an issue. Some applications may benefit from dynamic well models, in particular during start-up of wells. Start-up may occur quite often since many wells are shut-in from time to time due to maintenance or operational problems. Applications with long pipelines may also benefit from dynamic pipeline models provided the dynamics are important for optimal performance.

6. CONCLUSIONS

This paper argues that Dantzig-Wolfe Decomposition is well suited for the well allocation and routing problem in the upstream industries. There are several reasons for this. DWD clearly outperforms a global method. DWD has several similarities to LR. However, as the results show, DWD gives better performance than LR in all relevant cases tested herein. This is due to more efficient updating of the dual variables. Furthermore, an error bound on the solution of the production optimization problem can easily be computed. This is clearly information of interest to any user. Finally, the algorithm is efficient and can be parallelized for even higher efficiency.

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