

# Stability of timber beams and columns

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## Summary

Stability of beams, columns and beam-columns, as solved by the current versions of two timber codes (the Norwegian code and Eurocode 5), is reviewed. Changes are suggested for the combined bending and compression action in beam-columns, based on numerical analyses, both linearized buckling and geometrically non-linear analyses. Bracing of beams prone to lateral, torsional buckling is also investigated, and for simple beams the effect of the bracing and its required stiffness, as a function of where in the beam section the bracing is applied, is demonstrated.

**Keywords:** Stability, linearized buckling, lateral torsional buckling, non-linear analysis, timber codes

## 1. Introduction

Stability is often a sticky point in structural design, and timber structures are no exception. Lateral, torsional buckling is particularly difficult, and most building codes offer little assistance. In the real world, instability results in excessive displacements which in turn produce high bending stresses that will cause material failure. There are basically two different approaches to this non-linear problem: 1) the simplified method specified by the codes, in which the response, determined by ordinary linear theory, is modified by correction factors ( $k$ -factors) that will, in an approximate way, account for the non-linear effects, and 2) more accurate non-linear response analysis, including geometrical imperfections, that will account directly for the (geometrical) non-linear effects.

The  $k$ -factors of the simplified approach are determined on the basis of the critical (buckling) loads determined by classical methods for a few simple, but very basic and idealized cases, like the simply supported (Euler) column and beam. Slenderness ratios and thus buckling (or effective) lengths are key parameters for the determination of the  $k$ -factors. The design of individual components of more complex structures by the simplified method therefore depends to a large extent on the engineer's ability to estimate appropriate values for the equivalent buckling or effective lengths of the members.

The second method, which most codes will permit but not address in much detail, requires fairly complex computational procedures that can only be solved by means of computers. 2D models can provide adequate results for certain problems, but since most stability problems involve both in-plane and out-of-plane displacements, only 3D models can provide complete solutions [1]. Reference [2] describes a comprehensive investigation into the stability problem of beam type timber structures. The first part of the work is concerned with the development of a consistent non-linear 3D beam element, based on second order theory and with internal geometric stiffness. The element is implemented into a co-rotated, geometrically non-linear formulation in a special purpose program, Cfem. Both linearized buckling and geometrically non-linear problems can be solved for general 3D models of frame type structures. Each beam member is modelled by a series of straight elements with constant cross section properties. Curved members and members with varying cross section are easily

approximated, and the approach also facilitates inclusion of shape imperfection. All distributed loading is lumped into equivalent, concentrated loads at nodal points that can be located eccentrically in relation to the beam axis. Fairly general boundary conditions, including springs, can be specified at any node.

The Cfem program has been thoroughly tested and found to be a robust and flexible tool. With this type of program it is possible to design timber structures, including all bracing members, without resorting to any kind of correction factors. For such tools to become generally accepted design tools, we need reliable failure criteria for the timber material, appropriate material stiffness properties and automatic or semi-automatic definition of shape imperfections. Improved models for the semi-rigid behaviour of the various joints in timber structures would also enhance the quality of results and encourage the use of such tools.

While the ultimate goal is to develop rational methods that model and analyse the real problem without resorting to correction factors sensitive to engineering knowledge and judgement, tools like Cfem can also be used to improve the simplified methods. In [2] a good many (approximate) formulas for simple beams have been developed, and some existing ones have been corrected. In this paper we will focus on some of this work, presenting results primarily for beam-columns. The findings are compared with the specifications of the codes, and for some of these specifications changes are suggested. The program is also used to look into the problem of bracing of beams prone to lateral torsional buckling.

## 2. Instability and the codes

In this section we summarize briefly how instability is handled in two timber codes, the current Norwegian code, NS 3470-1[3] and the current proposal for Eurocode 5 or EC5 [4]

### 2.1 Columns - flexural buckling

Buckling of a pure compression member is here referred to as *flexural buckling*. The Norwegian code requires that such a member, called a column, shall satisfy the following condition:

$$\frac{\sigma_{c0f}}{k_{\lambda} \cdot f_{c0d}} \leq 1.0 \quad (1)$$

$\sigma_{c0f}$  and  $f_{c0d}$  are the design compressive stress and strength along the grain, respectively.  $k_{\lambda}$  is the correction factor accounting for buckling. It is based on a certain imperfection and it depends primarily on the slenderness ratio  $\lambda = l_k/i$  taken in the ‘weak’ direction.

EC5 includes bending in its section about columns and thus has no provision for pure compression members. The conditions to satisfy are:

$$\frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \quad (2.a)$$

$$\frac{\sigma_{c,0,d}}{k_{c,y} f_{c,0,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} \leq 1 \quad (2.b)$$

If we disregard the bending terms (*i.e.*  $\sigma_{m,y,d} = \sigma_{m,z,d} = 0$ ) Eqs. (2a and b) are equivalent to Eq. (1). The factors  $k_{c,y}$  and  $k_{c,z}$  are similar to  $k_{\lambda}$ , but not identical. For solid timber and glue laminated timber the  $k_m$  factor is 0.7 for rectangular sections (otherwise it is 1.0).

### 2.2 Beams - lateral torsional buckling

The buckling of beams subjected to bending about the ‘strong’ axis is treated in a very similar way by the two codes. NS 3470-1 requires the following condition to be satisfied.

$$\frac{\sigma_{mf}}{k_h \cdot k_{vipp} \cdot f_{md}} \leq 1.0 \quad (3)$$

where  $k_h$  is a height factor and  $k_{vipp}$  is a buckling factor dependent on the relative slenderness for bending,  $\lambda_m = \sqrt{f_{mk}/\sigma_{mc}}$ , where  $\sigma_{mc}$  is the critical bending stress according to the classical theory of stability, with 5-percentile stiffness values. For bending about two axes the following condition must also be satisfied

$$\frac{\sigma_{myf}}{k_{hy} \cdot k_{vipp} \cdot f_{md}} + \frac{\sigma_{mzf}}{k_{hz} \cdot f_{md}} \leq 1.2 \quad (4)$$

EC5 requires the bending stress to satisfy the condition

$$\sigma_{m,d} \leq k_{crit} f_{m,d} \quad (5)$$

where the factor  $k_{crit}$  is, for all practical purposes, the same as  $k_{vipp}$ . Hence, with the height factor included in the design bending strength ( $f_{m,d}$ ), Eqs. (3) and (5) are the same. However, EC5 does not seem to have an equivalent to Eq. (4). Both NS 3470-1 and EC5 specify a value of 1.0 for  $k_{vipp}$  and  $k_{crit}$ , respectively, if the slenderness ratio  $\lambda_m$  is below 0.75.

### 2.3 Beam-columns - combined flexural and lateral torsional buckling

For combined compression and bending, NS 3470-1 requires the following condition to be satisfied:

$$\frac{\sigma_{c0f}}{k_\lambda \cdot f_{c0d}} + \left( \frac{\sigma_{myf}}{k_{hy} \cdot k_{vipp} \cdot f_{md}} + \frac{\sigma_{mzf}}{k_{hz} \cdot f_{md}} \right) \cdot \frac{1}{\left( 1 - \frac{k_\lambda}{k_{eu}} \cdot \frac{\sigma_{c0f}}{f_{c0d}} \right)} \leq 1.0 \quad (6)$$

This equation assumes the y-axis to be the strong axis of the cross section. The factor  $k_{eu}$  is defined as  $k_{eu} = \sigma_{euk}/f_{c0k}$  where  $\sigma_{euk}$  is the Euler stress ( $= \pi^2 E_{0k}/\lambda^2$ ). Equation (6) clearly recognizes the interaction between compression and bending.

EC5 however, recognizes only a very limited (and crude) interaction between compression and bending. It states: ‘For members subjected to combined axial force and bending, column stability and lateral torsional buckling stability may be verified independently in accordance with 6.3.2 and 6.3.3, respectively’. Sections 6.3.2 and 6.3.3 correspond to Eqs. (2) and (5), respectively.

## 3. Compression and bending interaction

The position taken by Eurocode 5 on the beam-column problem is somewhat surprising, as it seems unlikely to expect that interaction between compression and bending is adequately described by Eqs. (2a and b). This view is strongly supported by results obtained in [2]. Here we use the Cfem program to demonstrate this on a couple of typical beam-column problems.

### 3.1 The problem

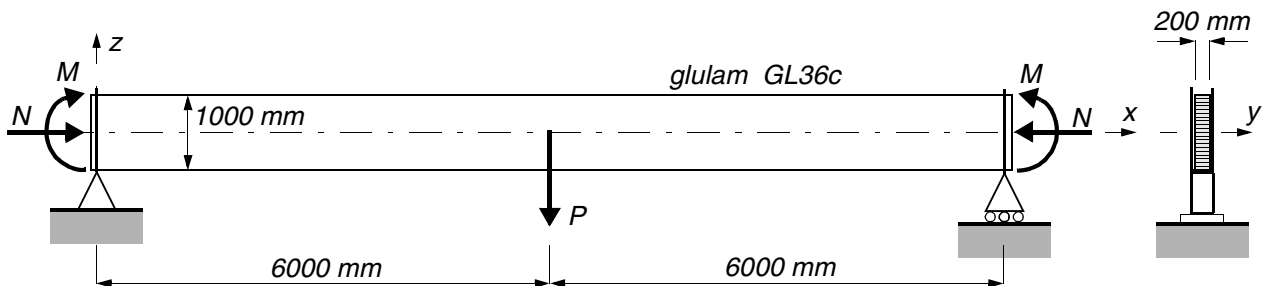


Figure 1 Test beam-column

The problem analysed is shown in Figure 1. A simply supported beam of glulam GL36c with dimensions 200×1000×12000 mm is considered. At the two supports, rotation about the beam axis (x) is prevented by a ‘fork’-support. Material properties for strength and stiffness are taken from the

Norwegian code (NS 3470-1). Two load situations are considered:

- 1) the beam is subjected to a constant moment  $M$  about the ‘strong’ axis ( $y$ ) and a centrally applied axial compression  $N$ , and
- 2) the beam is subjected to a concentrated load  $P$  at mid-span, applied at the area center of the cross section, and a centrally applied axial compression  $N$ .

### 3.2 The analyses

Both linearized buckling and geometrically non-linear analyses are carried out. In all cases the beam is analysed by a model in which it is subdivided into 40 elements of equal length. With the newly developed element this is a very fine mesh indeed, and it eliminates all discretization errors.

*Linearized buckling* - The critical values,  $N_{cr}$  and  $M_{cr}$ , of the axial force  $N$  (when acting alone) and of the (maximum) bending moments due to  $M$  (acting alone) and  $P$  (acting alone) are determined. In order to determine a point on the interaction curve, the beam is subjected to a *constant* value of  $N$  (a fraction of  $N_{cr}$ ) and the critical value of the maximum bending moment (due to the applied  $M$  or  $P$ ) is determined by solving the linearized buckling (eigenvalue) problem in which only the bending moment is allowed to vary (with the eigenvalue or buckling factor) - the geometric stiffness due to  $N$  is included in the ordinary stiffness matrix. The values of  $N$  and  $M$  are normalized by the critical values  $N_{cr}$  and  $M_{cr}$ , respectively. Sixteen such points have been determined and the results are shown in Figures 2 and 3 where the points are connected by straight lines.

*Non-linear analysis* - With an imperfection, in the shape of the first flexural buckling mode (as determined with  $N$  acting alone) and an amplitude of  $L/500$  (i.e. 24 mm), and a simple failure criteria

$$\frac{\sigma_m}{f_{mk}} + \frac{\sigma_{c0}}{f_{c0k}} > 1.0 \quad (7)$$

the critical values of the axial force ( $N_{cr}$ ) and the bending moments ( $M_{cr}$ ), when acting alone, are determined through a geometrical non-linear analysis. These critical values are about 10% lower than the corresponding values from the linearized buckling analyses. Points on the interaction curves are determined in much the same way as outlined above: for certain constant values of  $N$  (a fraction of  $N_{cr}$ ) the failure value of  $M$  is determined through a non-linear analysis. Again the values of  $N$  and  $M$  are normalized by the critical values  $N_{cr}$  and  $M_{cr}$ , respectively. The results, based on nine points, are shown in Figures 2 and 3.

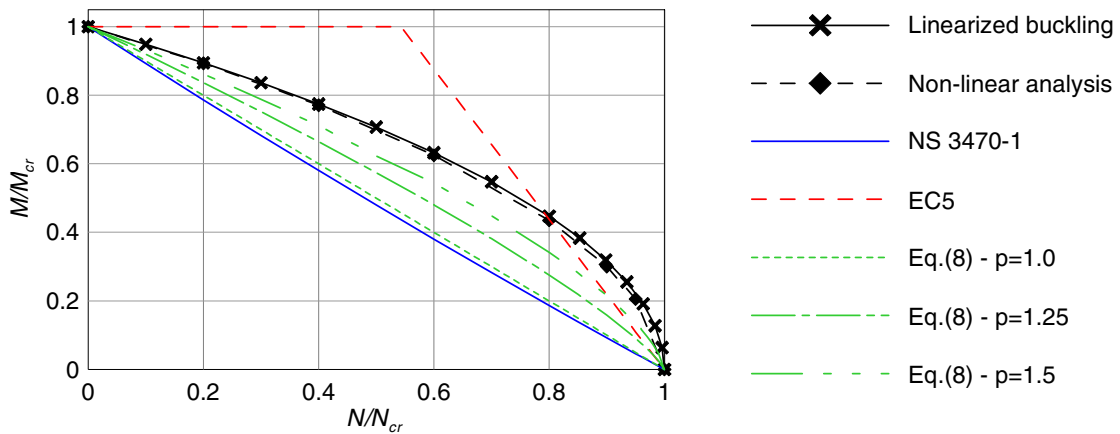


Figure 2 Combined bending and compression for the beam-column subjected to  $M$  and  $N$

### 3.3 Discussion and suggestions

Figures 2 and 3 show that the interaction curves obtained by linearized buckling analyses are very similar to those obtained by geometrically non-linear analyses. The figures also include curves for how the two codes would treat these problems. We see that while Eq. (6) of NS 3470-1 is very

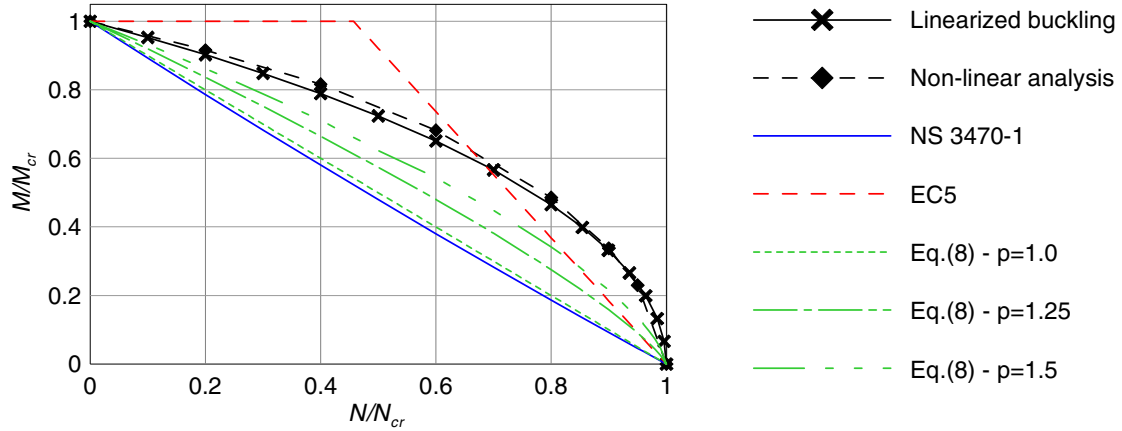


Figure 3 Combined bending and compression for the beam-column subjected to  $P$  and  $N$

conservative, EC5 through Eqs. (2) and (5) will, for certain combinations of  $N$  and  $M$  overestimate the capacity by as much as 30%. More results for other beam geometries, loads, load applications and boundary conditions are reported in [2], and they all show much the same picture. It should be noted that *all* results are obtained with characteristic values for the material properties.

We believe that, based on the numerical results, the beam-column problem should be treated by an interaction formula of the type

$$\left(\frac{N}{N_{cr}}\right) + \left(\frac{M}{M_{cr}}\right)^p = 1.0 \quad (8)$$

In Figures 2 and 3 curves are included for three values of  $p$ ,  $p = 1$  (linear interaction),  $p = 1.25$ , and  $p = 1.5$ . The prudent approach is perhaps to adopt the most conservative value, *i.e.*  $p = 1.0$ . In line with this we would suggest that NS 3470-1 drop the last, ‘amplifying’, term on the left-hand side of Eq. (6). This will improve the results slightly and it will significantly simplify the formula. For Eurocode 5 we would suggest that section 6.3.2 be restricted to pure compression members (as the heading indicates), and that the bending terms of Eqs. (2a and b) are dropped. The section on beams (6.3.3) would remain as it is, but a new section (6.3.4) on ‘Combined bending and compression (beam-columns)’ should be included, in which the following two conditions should be satisfied:

$$\frac{\sigma_{c,0,d}}{k_{c,z}f_{c,0,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} + k_m \frac{\sigma_{m,y,d}}{k_{crit}f_{m,y,d}} \leq 1 \quad (9.a)$$

$$\frac{\sigma_{c,0,d}}{k_{c,y}f_{c,0,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} + \frac{\sigma_{m,y,d}}{k_{crit}f_{m,y,d}} \leq 1 \quad (9.b)$$

These conditions assume  $y$  to be the strong axis of the cross section. The proposed changes would bring EC5 into the conservative realm for all stability problems of individual members without adding much, if any complexity to the procedures.

#### 4. Bracing of beams

The load bearing capacity of beams prone to lateral torsional buckling may be considerably increased by restraining one or more points from lateral displacement by bracing. In most practical design guides, including some codes, it is recommended and sometimes also required that only bracing placed at the extreme compressive fibres is effective in this respect. In order to investigate the effectiveness of bracing as a function of its point of application, a large number of linearized buckling analyses were carried out on simple glulam beams [2]. Different beam geometries, loading and boundary conditions were considered. Here we present some of the results, all of which were obtained for a simply (‘fork’) supported beam with dimensions  $140 \times 600 \times 8000$  mm, subjected to 1) a constant moment  $M$  and 2) a concentrated load  $P$  acting at mid-span and applied at the cross section center, see

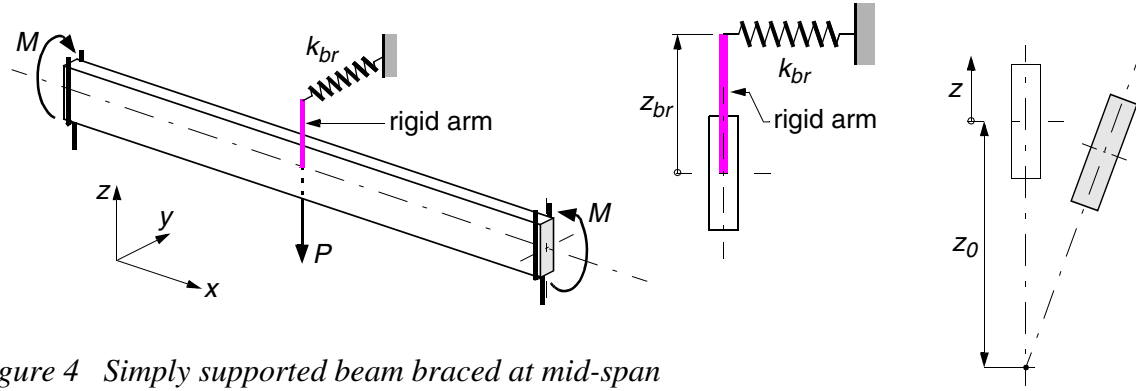


Figure 4 Simply supported beam braced at mid-span

Figure 4. For each loading the second lowest buckling load for the unconstrained beam (corresponding to two half waves) is determined,  $M_2$  and  $P_2$ , respectively.

Buckling of the beam implies both translation and rotation of the beam section, and, for the classical case of a simply supported beam subjected to a constant moment, it is possible [2] to derive an expression for the center of rotation  $z_0$  (Figure 4),

$$z_0 = -(L/\pi) \sqrt{(GI_t)/(EI_z)} \quad (10)$$

Next, a bracing spring in the transverse (y-) direction is applied eccentrically, that is, at a distance ( $z_{br}$ ) from the beam axis at mid-span, see Figure 4. Figure 5a shows the results from a large number of linearized buckling analyses in which the distance  $z_{br}$  has been varied. The loading is a constant moment, and the results in Figure 5a apply to an *infinitely stiff* spring. The results are normalized with

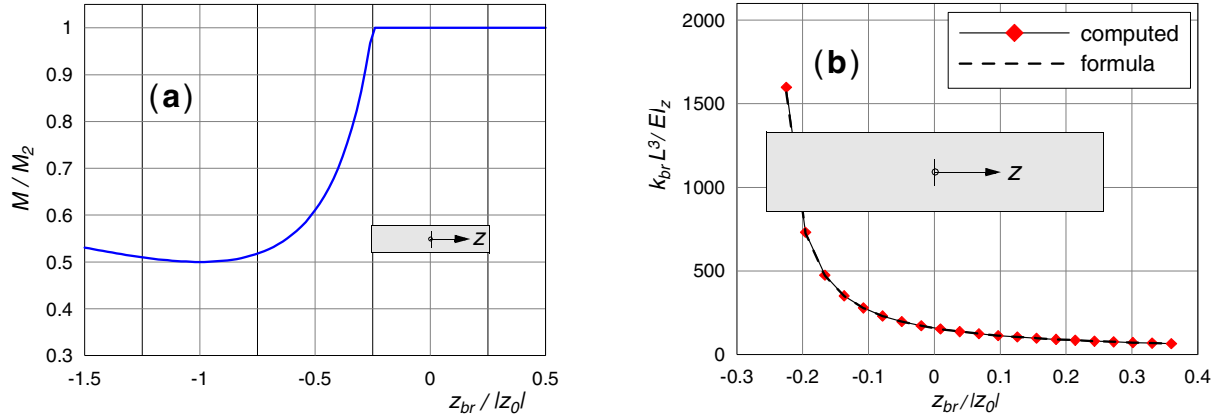


Figure 5 Simply supported beam subjected to a constant moment  $M$  - **a)** effect of position of stiff bracing and **b)** necessary stiffness to force beam to buckle in two half waves

respect to  $|z_0|$  and  $M_2$ , respectively, and the cross section, drawn to scale, is included in the figure. It is seen that an infinitely stiff spring will force the beam to buckle in two half waves (for a critical moment equal to  $M_2$ ) for any position of the spring within the beam height. We also see that the only point where the spring has no beneficial effect is when it is placed in the center of rotation ( $z_{br} = z_0$ ), for which the critical moment ( $M_{cr} = 0.5M_2 = M_1$ ) is equal to that of the unconstrained beam. Hence, the numerical analyses confirm the position of the center of rotation. In [2] it is shown that the results for two other beam geometries coincide more or less completely with the curve in Figure 5a.

How stiff must the bracing spring be in order to force the beam to buckle in two half waves? In Figure 5b this minimum stiffness is shown as a function of the spring position. Since these results involve iteration, Figure 5b is the result of more than 2000 individual analyses. The numerical results have been fitted to the following approximate formula for the required spring stiffness:

$$k_{br} = \frac{1}{0.0253(z/|z_0|) + 0.00633} \cdot \frac{EI_z}{L^3} \quad (11)$$

Figure 5b shows very good agreement with the numerical results. We also see that we need roughly to double the bracing stiffness when we move the point of application from the top fibre to mid-height, but the required stiffness is still quite moderate.

Figure 6 shows the same results for the beam subjected to a concentrated load  $P$  at mid-span.

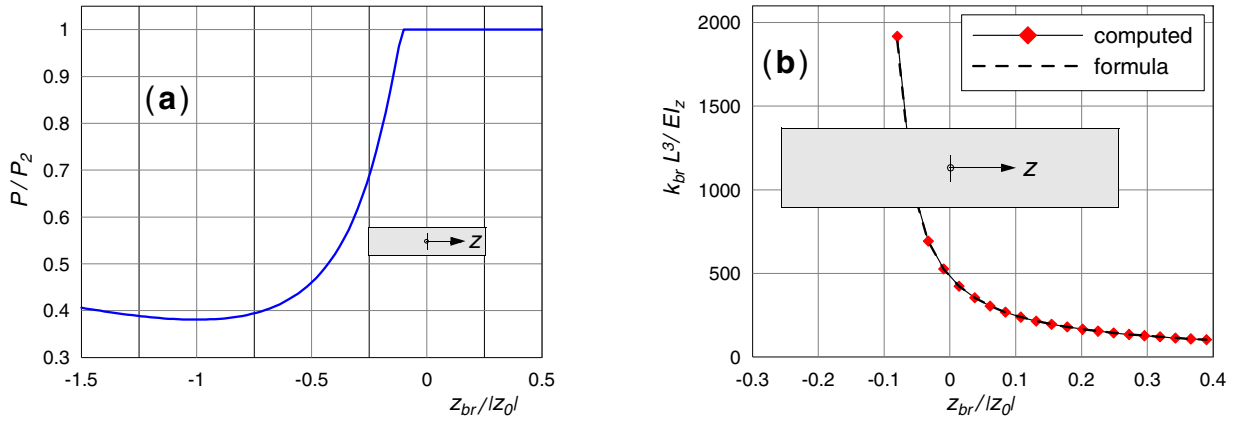


Figure 6 Simply supported beam subjected to a concentrated load  $P$  - **a)** effect of position of stiff bracing and **b)** necessary stiffness to force beam to buckle in two half waves

Expressions for  $z_0$  and  $k_{br}$ , determined numerically, are

$$z_0 = -(L/3.35)\sqrt{(GI_t)/(EI_z)} \quad \text{and} \quad k_{br} = \frac{I}{0.0195(z/|z_0|) + 0.00209} \cdot \frac{EI_z}{L^3} \quad (12a \text{ and } b)$$

In this case it is not possible to force the beam to buckle into two half waves if the bracing is placed below a point located at approximately  $h/4$  from the bottom, regardless of the spring stiffness. If the bracing is placed at mid-height it needs to be about twice as stiff as in the case of constant moment loading, and about three times as stiff as if it was placed at the extreme top fibre, in order to have its maximum effect (*i.e.* to force the beam to buckle in a higher mode). The case of a uniform load will, as shown in [2], exhibit results that lie between the two sets of results reported here (in fact, closer to Figure 5).

From the results reported here it seems fair to state that all bracing restricting lateral displacement is fully effective if placed anywhere between mid-height and the extreme compression fibres. However, the further away the bracing is placed from the extreme compression fibres, the stiffer it needs to be, as much as a factor of 3. With this in mind, bracing at the middle of the section height should be quite adequate, and the required stiffness is easily attainable.

## 5. References

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